Universality Explained

Alexander Franklin

April 16, 2016

Abstract

It is commonly claimed, both by physicists and philosophers that the universality of critical phenomena is explained through particular applications of the Renormalisation Group (RG). This paper seeks to clarify this explanation.

The derivation of critical exponents proceeds in two ways: (i) via a real-space and (ii) via a momentum-space application of the RG. Following Mainwood (2006) I argue that these approaches ought to be distinguished: while (i) fails adequately to explain universality, (ii) succeeds in the satisfaction of this goal.

(i) depends on various extensions to the Ising model. These serve as archetypes of the different universality classes. I emphasise that the derivation does not take diverse systems and justify their inclusion in each universality class, rather universality is assumed and the critical exponents are obtained for each class from its archetype alone.

(ii) starts with an effective Hamiltonian which abstracts away from the details of different physical systems. It can be shown that the addition of various operators to this Hamiltonian would be irrelevant to the derived values of the critical exponents; this implies that multiple Hamiltonians belong to the same universality class. As such, universality is explained by the general applicability of the effective Hamiltonian.

I further claim that we have good reason to believe that a reductive explanation of universality could be formulated; this follows from the explanatory strategy clarified in previous sections. I argue that the possibility of a reductive explanation undermines claims in Batterman (2014) and Morrison (2014) that the RG explanation of universality is irreducible. In addition, this may provide a paradigm example of a reductive explanation of multiple realisability.

Contents

1 Introduction

'Universality' is the technical term for a striking kind of multiple realisability. Physical systems exhibit phase transitions: abrupt variation in macroscopic behaviour such as the transformation from water to steam. For fluids, such clearly defined liquid-gas transitions happen only below a certain temperature known as the critical point. Above that temperature there is no clearly defined transition.

The universality in question corresponds to the behaviour of systems as they approach this critical point. It turns out that such behaviour can be very well described by power laws of the form $a_i(t) \propto t^{\alpha}$ where t is proportional to the temperature deviation from the critical temperature.[1] The striking phenomenon is that physical systems can be categorised into universality classes according to their behaviour as they approach the critical point: members of the same class have identical critical behaviour – the

[1] E.g. the specific heat (c) scales as $c \sim \alpha^{-1}(t^{-\alpha})$ as $T \rightarrow T_c$ and $B = 0$. $t = \frac{T-T_c}{T_c}$.

same set of critical exponents $\{\alpha, \beta, ...\}$ for several power laws – while they may have radically diverse microphysical structures.

A paradigm example of universality is that the liquid-gas critical phase transition and the (uniaxial) ferromagnetic-paramagnetic critical phase transition share critical exponents. Both of these types of systems may be described by equivalent power laws as they transition from certain ordered states (liquid or magnetised respectively) to critical states. These systems are examples of the 3D Ising universality class.

Hundreds of papers have been published in Physics journals over the last fifty years on this topic. On the one hand a great deal of experimental evidence is available which classifies many different physical systems into a few universality classes, and finds the critical exponents for these classes to ever greater accuracy; see Sengers and Shanks (2009) and references therein. On the other, theoretical work is continually under way to refine and develop the theoretical models for each universality class; see Pelissetto and Vicari (2002). It is now the case that both through computer modelling (Monte Carlo simulations) and through field-theoretic derivations (using perturbation theory) critical exponents derived match very closely those discovered empirically.

How is universality to be explained? This question generalises to debates concerning the explanation of multiply realised phenomena: how ought one to explain instances of common behaviour between systems which have diverse microstructures? A standard response is that we ought to discover the features of such systems which are sufficient for such behaviour. If those features are shared between the various systems then we have an explanation of the common behaviour, if not one ought to conclude that the common behaviour is coincidental.[2]

Batterman (2014) asserts that an adequate explanation of universality must show how heterogeneous features are irrelevant. This demand is clearly satisfied by the identification of common features sufficient to derive the common phenomenology. If details are identified sufficient for some behaviour to occur then other aspects of the system under consideration will be irrelevant to that occurrence. The discovery of sufficient common features is thus the standard for an adequate explanation to which I appeal throughout this paper.

[2]Some instances of apparent commonality might fail to qualify as true instances of commonality, in such situations the coincidence will not be surprising.

In the following I outline the background physics (§2) and analyse the explanation on offer by the momentum-space and the real-space approaches to the Renormalisation Group (RG); see §3.1 and §3.2 respectively.[3] In the physics literature it is standard practice to distinguish these approaches; I will argue, following Mainwood (2006) that the distinction is also significant when assessing the RG explanation of universality. I, like Mainwood, endorse the momentum-space explanation of universality while arguing that the real-space explanation is generally inadequate. However, my reasons for believing this are quite different from those adduced by Mainwood.

The distinction between these explanatory approaches and the adoption of the momentum-space approach implies that each member of each universality class is distinguished by a set of 'irrelevant operators'. This has the consequence that, contra the primary claims of Batterman (2014) and Morrison (2014) a reductive explanation of universality is available; see §4.1.[4]

Close attention to the physics thus has a philosophical pay-off: a schema is outlined for deriving the momentum-space RG explanation from the microscale Hamiltonians ascribed to each individual system. This will be of interest to philosophers with a stake in the emergence-reduction dialectic. Batterman (2014) argues, *pace* Sober (1999), that universality does in fact pose a challenge to reductionism; in §4 I outline the philosophical dialectic. Batterman's argument rests on the claim that an explanation of universality (which he regards as a paradigm example of multiple realisability) is unavailable. As such, my claims here are of central relevance to that debate.

2 The Physics

The following two sections involves some technical detail. What do I show in non-technical terms? That the two approaches to the RG provide differ-

[3] Although more generally momentum-space and real-space depictions are equivalent – simply related by a Fourier transform – in the current context these are labels used to refer to different derivations of the critical exponents.

[4] This involves the bracketing of claims about the emergence implied by the infinitary idealisations which are appealed to in the renormalisation group account of critical phenomena; although I do not think that this issue has been satisfactorily resolved, discussion may be found in Batterman (2005), Butterfield and Bouatta (2011) and Callender and Menon (2013).

ent putative explanations of universality and that, as such, they ought to be distinguished.

I conclude that the real-space approach allows for the derivation of universality based only on a representative model for each universality class. A model is not provided for each member of the same class and it is not demonstrated that the details which distinguish each member of the same class are irrelevant to that system's critical behaviour. Thus universality is *not explained but assumed*: no justification is given for the application of the single model to the other members of the class. This conclusion is reached through consideration of the models and a sketch of the RG methods by which the critical exponents are derived for each such model.

In §3.2 I consider a possible response to this assumed-not-explained objection: this involves the claim that liquid-gas systems and uniaxial magnetic systems have common behaviour because of a structural mapping (the lattice-gas analogy) between them. I express doubts about this reasoning, in addition I argue that the behaviour of the broad range of systems which feature universality is not thus explained.

I also describe the momentum-space RG approach. I argue that only the momentum-space RG (because of its use of a renormalisable Hamiltonian) has the tools to describe the commonalities in the various systems sufficient to their common behaviour. This explanation proceeds via demonstration of the genericity of the Hamiltonian under consideration. But this explanation is somewhat different to the explanatory sketch on offer in some physics and philosophy texts. The standard account implicitly depends on physics which has not been worked out, as such it includes certain technical lacunae. In §3.1 I make these lacunae explicit and adduce reasons to consider the momentum-space RG explanation nonetheless adequate.

The momentum-space explanation of universality identifies a particular Hamiltonian description which applies to all systems within the same class, and thus via the use of RG methods derives their critical exponents. The explanation depends on the demonstration that this Hamiltonian applies to these various systems (the common feature) and the RG methods. This grounds Batterman's (2000,2014) claim that the explanation relies on the behaviour of the Hamiltonians around the fixed point and relates it to the underlying description of these physical systems.

2.1 The Models

It turns out that the critical behaviour of the different universality classes can be derived from a range of simple model systems. I briefly describe the Ising model, and its extension to the n-vector model which defines a broad range of models classified according to their values for two variables. This model is crucial to understanding the real-space RG, and is abstracted to provide the basis for the momentum-space RG. Microphysical models are not defined for multiple members of the same universality class, rather a representative model is used for each class.

Martin Niss (2005) describes the early history of the Lenz-Ising model.[5] This history demonstrates that the Ising model was specifically designed to represent the physical characteristics of magnetic systems rather than the broader range of systems which display critical phenomena. Niss observes that it was commonplace in the 1920s to model magnetic materials as composed of a lattice of numerous micromagnets – often idealised as compass needles – which mutually interact. The model was proposed to represent the transition between the ferromagnetic and paramagnetic states of certain materials.[6] The major innovations due to Lenz and Ising were to define a particular interaction between neighbouring micromagnets and to restrict their possible orientations to a discrete range. This latter assumption arose out of a combination of empirical data, knowledge of the structural and symmetry properties of solid matter and considerations from early quantum mechanics.

In modern formulations the Ising model is described as an array of spins. It consists of a D-dimensional cubic lattice with $\{\mathbf{e_i}\}$ basis vectors with sites labelled $\mathbf{k} = (k_1\mathbf{e_1}, ..., k_D\mathbf{e_D})$. At each site there is a spin variable $\sigma_\mathbf{k} \in \{-1, 1\}$, though in extensions to this model the spin variable can take a greater range of values. A Hamiltonian is defined:

$$\mathcal{H} = -J \sum_{\mathbf{k},\mathbf{k}+\boldsymbol{\mu}} \sigma_\mathbf{k}\sigma_{\mathbf{k}+\boldsymbol{\mu}} - B \sum_\mathbf{k} \sigma_\mathbf{k} \tag{1}$$

The coupling constant J takes a positive value and is assumed to be

[5] I will henceforth refer to it as the 'Ising model' – as it is generally known – although Lenz and Ising jointly proposed it in papers in 1920 and 1924 respectively.

[6] The Ising model also predicts the spontaneous magnetisation below the critical temperature, though this was not discovered until Peierls did so in 1936.

independent of all variables other than the system volume. The Ising model interaction is generally defined over nearest, or next-nearest neighbours, thus $\boldsymbol{\mu}$ is a lattice vector which takes any vector to the relevant neighbour in the positive direction. B is an external magnetic field.

The Hamiltonian of a system corresponds to the energy of the system in a particular configuration, thus we see (as the phenomenology suggests) that the Ising Hamiltonian will take a lower value when the spins are aligned, and a higher value when spins are disordered. The ferromagnetic-paramagnetic transition can be defined over this lattice as the transition from the spin configuration with all spins aligned to that where there is no general correlation between the spin directions. This transition will take place at the Curie temperature (T_c). In 1944 Lars Onsager published a paper which derived the specific heat of a two dimensional Ising model in the absence of an external magnetic field. He demonstrated that this system will display power law behaviour with a particular critical exponent. However, despite much effort, no-one has succeeded in an analytic derivation of critical behaviour for any three dimensional model.

Behaviours characteristic of systems approaching T_c are termed 'critical phenomena' and it is with respect to the power laws which describe such behaviour that universality can be observed. Current mathematical procedures to describe such behaviour involve the Renormalisation Group (RG) which I describe below. First I note the n-vector model which generalises the Ising model to various universality classes. As Stanley (1999, p. S361) notes: "empirically, one finds that all systems in nature belong to one of a comparatively small number of such universality classes".

The n-vector model includes spins which can take on a continuum of states.

$$\mathcal{H}(d,n) = -J \sum_{\mathbf{k},\boldsymbol{k}+\boldsymbol{\mu}} \boldsymbol{\sigma}_{\mathbf{k}} \cdot \boldsymbol{\sigma}_{\boldsymbol{k}+\boldsymbol{\mu}} - B \sum_{\mathbf{k}} \boldsymbol{\sigma}_{\mathbf{k}} \tag{2}$$

Here, the spin $\boldsymbol{\sigma}_{\mathbf{k}} = (\boldsymbol{\sigma}_{\mathbf{k},1}, \boldsymbol{\sigma}_{\mathbf{k},2}, ..., \boldsymbol{\sigma}_{\mathbf{k},n})$ is an n-dimensional unit vector. The two parameters which determine the universality class are the system dimensionality d (which will determine the set of nearest neighbours) and the spin dimensionality n. The standard, 3D Ising model corresponds to $\mathcal{H}(3,2)$.

I now turn to a discussion of the renormalisation group derivation of critical exponents. A full exposition would require more space than we have here but I sketch the procedure below.[7] RG transformations are constructed to preserve thermodynamical properties of the system of interest (those derived from the partition function) while increasing the mean size of correlations. Thus, for example, the RG transformations take a ferromagnetic system towards the critical point (where the order parameter fluctuates wildly).

2.2 Momentum-space and Real-space Renormalisation

I mentioned above that there are competing methods for deriving the critical exponents for each universality class. These correspond to different RG approaches:

Real-space RG: Consider the Hamiltonian of a system on a lattice (e.g. in the Ising model). The higher energy interactions will probe the structure of the lattice, and in order to consider the system probed at a larger length-scale, we average over the higher energy contributions to the Hamiltonian. This can be done by increasing the effective lattice size and constructing a new Hamiltonian for a system on a larger lattice, see figure 2; this is sometimes referred to as 'coarse-graining' or 'zooming out'. This can be thought of as a blocking procedure, whereby some group of particles is replaced by one particle which represents the group through an average or suchlike.[8] On this model the RG flow represents the changes in parameters which leave the form of the Hamiltonian, and certain qualitative properties of the system unchanged (i.e. those which are derived from the partition function) while increasing the lattice size. Monte Carlo computer based methods allow for the derivation of the critical exponents from the n-vector Hamiltonian (equation (2)) via the real-space RG.

Momentum-space RG: The Hamiltonian (equation (7)) considered in this case is more abstract (technically it is a functional of the order parameter) and depends for its construction on Ising-type models – I discuss its derivation below. The calculation of this Hamiltonian for real systems involves integration over a range of scales and energies. The highest energy (smallest scale) cut-off (denoted Λ) corresponds to the impossibility of fluctuations on a scale smaller than the distance between the particles in the

[7] There are many textbooks and review articles which describe these techniques, see for example Binney et al. (1992), Cardy (1996) and Fisher (1998).

[8] A variety of acceptable blocking methods are discussed by Binney et al. (1992).

physical system. The RG transformation in this case involves decreasing the cut-off, thus increasing the minimum scale of fluctuations considered. This procedure is analogous to increasing the lattice size and will similarly generate a flow through parameter space designed to maintain the Hamiltonian form and qualitative properties of the system in question.[9]

At this stage I go into slightly more detail, the aim being to provide a clear sketch of the derivation of the critical exponents for each approach. The RG transformation $\mathcal{R}$ transforms a set of (coupling) parameters $\{K\}$ to another set $\{K'\}$ such that $\mathcal{R}\{K\} = \{K'\}$. $\{K^*\}$ is the set of parameters which corresponds to a fixed point (FP), defined such that the RG transformation will have no effect on the set of parameters transformed, as such $\mathcal{R}\{K^*\} = \{K^*\}$. If we assume that $\mathcal{R}$ is differentiable at the fixed point this leads us to a version of the RG equations.

$$K'_a - K^*_a \sim \sum_b T_{ab}(K_b - K^*_b) \tag{3}$$

$$\text{where } \; T_{ab} = \frac{\partial K'_a}{\partial K_b}\Big|_{K=K^*}$$

There are now two more steps before we can define relevance and irrelevance. Firstly we define the eigenvalues of the matrix T_{ab} as $\{\lambda^i\}$ and its left eigenvectors as $\{e^i\}$. Now we can define scaling variables which are linear combinations of the deviations from the fixed points: $u_i \equiv \sum_a e^i_a(K_a - K^*_a)$. By construction these scaling variables will transform multiplicatively near the fixed point such that $u'_i = \lambda^i u_i$. The second (trivial) step is to redefine the eigenvalues as $\lambda^i = b^{y_i}$ where b is the renormalisation rescaling factor and y_i are known as the renormalisation group eigenvalues.

If $y_i > 0$ then u_i is relevant; if $y_i < 0$, u_i is irrelevant; and if $y_i = 0$, u_i is marginally relevant. The relevant scaling variables will increase in magnitude after repeated RG transformations while the irrelevant scaling variables will tend to zero after multiple iterations. (The behaviour of the marginal scaling variables requires more analysis to determine.) Thus, given the Hamiltonian of one of our models one can define an RG transformation which will allow one to: (i) classify certain of the coupling parameters of the system in question as (ir)relevant to its behaviour near the

[9] A note on terminology: the momentum-space RG is so-called because the field-theoretic calculations involving diagrammatic perturbation theory are most easily performed in momentum-space, see Binney et al. (1992).

fixed point, (ii) extract the critical exponents from the scaling behaviour near the FP. Up to this point the description is generic.

The real-space RG depends on the application of a blocking transformation as depicted in figure 2 (p.18). It is required that the Hamiltonian form is stable across these transformations. Since the Hamiltonians are not renormalisable this involves the application of a transformation and subsequent truncation of the Hamiltonian.[10] This procedure is generally carried out using computer methods.

The momentum-space RG approach derives the critical exponents using diagrammatic perturbation theory which I do not have space to elaborate here. The Hamiltonian in this context is macroscopic and depends on the order parameter ($\boldsymbol{\phi}$) which – in the Ising model context – is a sum of the spins in a small region of volume δV at $\mathbf{x}$: $\boldsymbol{\phi}(\mathbf{x}) = \frac{\mu}{\delta V}\sum_{i\in\delta V}\sigma_i$. [11] We require that $a \ll \delta V \ll l$ where a is the physical lattice spacing and l is the dominant statistical length (often the correlation length). One can approach its construction from the Ising model as follows (see Klein, Gould, and Tobochnik (2012) for more details):[12]

Start with the Ising model (equation (1)); then postulate a form for the Helmholtz free energy $\mathcal{F}(\phi)$ of a system in contact with a heat bath. The terms in equation (4) correspond (a) to the interaction of the coarse grained Ising spins with an external magnetic field, (b) the interactions between

[10] It is these truncations which motivate Mainwood (2006)'s dismissal of the explanation on offer by the real-space RG. I discuss this further on p.13. See §3.2 for my distinct critique of the real-space RG explanation.

[11] The symbol ϕ is used to refer to the thermal average of the order parameter $\phi(\mathbf{x}, t)$. This quantity has a system-dependent definition. For example in liquid-gas transitions $\phi(\mathbf{x}) \equiv \rho(\mathbf{x}) - \rho_{\text{gas}}(\mathbf{x})$ where $\rho(\mathbf{x})$ is the average density in a volume centred on $\mathbf{x}$ i.e. is a fluctuating quantity and $\rho_{\text{gas}}(\mathbf{x})$ is the time-averaged density for the gas at the temperature at $\mathbf{x}$. Clearly, below T_c for gaseous systems and above T_c in general $\phi \approx 0$, but below T_c for liquid systems $\phi > 0$. Analogously at the ferromagnetic-paramagnetic transition, where the magnet is well modelled by the Ising model the order parameter is as defined above. Thus for ferromagnetic systems (at $T < T_c$) $\boldsymbol{\phi} \neq 0$ and for paramagnetic systems ($T > T_c$) $\boldsymbol{\phi} = 0$.

The order parameter is defined for many other systems: for the binary fluid $\phi(\mathbf{x}) = X'(\mathbf{x}) - X''$ where $X'(\mathbf{x})$ is the local molar density of one of the fluids and X'' its thermally averaged value when the fluids have separated; for Helium I - Helium II transitions the order parameter is $\psi(\mathbf{x})$ which is the quantum amplitude to find a particle of He II at $\mathbf{x}$; similarly for conductor-superconductor transitions where $\psi(\mathbf{x})$ is the quantum amplitude to find a Cooper pair at $\mathbf{x}$.

[12] There are many different derivations of this Hamiltonian which speaks to its generality. See Binney et al. (1992), Goldenfeld (1992) for some alternatives.

the coarse grained spins which depends only on the distance between the blocks and (c) an approximation of the entropy (using Stirling's approximation). $F = U - TS$.

$$\begin{aligned}\mathcal{F}(\phi) = &-\overbrace{B\int\phi(\mathbf{x})d\mathbf{x}}^{\text{(a)}}-\overbrace{\frac{1}{2}\iint J(|\mathbf{x}-\mathbf{y}|)\phi(\mathbf{x})\phi(\mathbf{y})d\mathbf{x}d\mathbf{y}}^{\text{(b)}}\\ &-k_BT\Big(\underbrace{\int[1+\phi(\mathbf{x})]\ln(1+\phi(\mathbf{x}))d\mathbf{x}+\int[1-\phi(\mathbf{x})]\ln(1-\phi(\mathbf{x}))d\mathbf{x}}_{\text{(c)}}\Big)\end{aligned} \tag{4}$$

Assuming $\phi(\mathbf{x})$ is small allows the logarithms to be expanded and truncated after the second order (on the assumption that the spin blocks only vary significantly over large distances). Using Parseval's theorem, expanding $J(|\mathbf{x}-\mathbf{y}|)$ in Fourier space, truncating after the second derivative, converting back to real-space and then integrating by parts leads to (b) becoming

$$\hat{J}(0)\int\phi(\mathbf{x})\phi(\mathbf{x})d\mathbf{x}+\frac{1}{2}R^2\int[\boldsymbol{\nabla}\phi(\mathbf{x})]^2d\mathbf{x} \tag{5}$$

This results in a modified version of equation 4:

$$\mathcal{F}(\phi)=\int d\mathbf{x}[R^2[\boldsymbol{\nabla}\phi(\mathbf{x})]^2+\epsilon\phi^2(\mathbf{x})+\phi^4(\mathbf{x})-B\phi(\mathbf{x})] \tag{6}$$

This is the Landau-Ginzburg free energy, where $R^2 \propto \int \mathbf{x}^2 J(|\mathbf{x}|)d\mathbf{x}$. This has the same form as the Landau-Ginzburg-Wilson (LGW) Hamiltonian (equation (7)).[13] I will not discuss the few remaining steps as the physical underpinnings have been outlined. The form changes only slightly when it is read as a functional integral and the system is considered in the absence of an external magnetic field ($B = 0$). The integral is generalised to dimension d.

$$\mathcal{H}=\int \mathrm{d}^d\boldsymbol{x}\Big[\frac{1}{2}\zeta^2|\boldsymbol{\nabla}\boldsymbol{\phi}(\mathbf{x})|^2+\frac{1}{2}\theta|\boldsymbol{\phi}(\mathbf{x})|^2+\frac{1}{4!}\eta|\boldsymbol{\phi}(\mathbf{x})|^4\Big] \tag{7}$$

Note however that the LGW Hamiltonian is not the Ising model effective Hamiltonian. This latter object is more complicated, however it is

[13] In Statistical Mechanics $\mathcal{F}=\frac{\mathrm{Tr}\{He^{-\beta H}\}}{Z}=\langle H\rangle$.

demonstrated in Binney et al. (1992, Appendix K), (and is plausible given its derivation) that equation (7) is a good approximation to a truncated form of the Ising Hamiltonian near the critical point.

The construction of equation (7) is quite different from equations (1-2). It builds on these models but abstracts from them. More details can be found in (e.g.) Fisher (1974). There he demonstrates the field theoretic methods which allow one to derive expressions for the critical exponents as functions of d and n, see equation (8) for the first few terms of the exponent α; this will give a value for various universality classes. This derivation depends on the functional integration of the LGW Hamiltonian over all functions $\boldsymbol{\phi}(\boldsymbol{x})$.

$$\alpha = \frac{4-n}{2(n+8)}(4-d) + \frac{(n+2)^2(n+28)}{4(n+8)^3}(4-d)^2 + ... \tag{8}$$

Crucially, it can be shown that the addition of certain terms to the LGW Hamiltonian will lead to irrelevant contributions which do not affect the values for critical exponents describing the approach to a given fixed point. In Binney et al. (1992, Ch.14) the criteria for relevance and irrelevance are derived. An operator O_p is relevant if $p - d(p-2)/2 > 0$ and irrelevant if $p - d(p-2)/2 < 0$ where d is the dimension of the system under investigation.[14] O_p specifies the power of ϕ under consideration. It is formally defined as follows:

$$O_p \equiv \int \mathrm{d}^d\boldsymbol{x}\, \lambda_p \sum_{m=0}^{p/2-1} (-1)^m \frac{C_m}{(p-2m)!}\phi^{p-2m} \tag{9}$$

$$\text{where} \quad C_m \equiv \frac{1}{2^m m!}\Big(\int^{\Lambda} \frac{\mathrm{d}^d\boldsymbol{q}}{\zeta^2 q^2}\Big)^m$$

This serves to establish that for the LGW Hamiltonian, for $d = 3$, any O_p with $p > 6$ will be irrelevant at the appropriate fixed point.[15] This is an

[14]Note that the Ising-type Hamiltonians used in the real-space RG approach are not renormalisable, as such criteria for relevance and irrelevance of additions to those Hamiltonians cannot be specified in this generality.

[15]Odd powers of ϕ are generally excluded for reasons of symmetry. For $d = 3$ it can be established perturbatively (at least to low orders) that O_6 is also irrelevant.

important result for the discussion in the remainder of this paper. Its generality depends on the justification for the applicability of the LGW Hamiltonian to various models. As we will see in what follows this will depend in part on the order parameter assigned to each member of each universality class.

The theory behind this result is relatively involved, but the idea is simple: the LGW Hamiltonian is renormalisable. This means that applying an RG transformation to the Hamiltonian will not add terms which cannot be absorbed into the parameters ζ, θ, η in equation (7) (by contrast the real-space RG will need to be truncated after each RG transformation). Thus the Hamiltonian is in some sense scale-invariant: its renormalisability means that it is independent of the details of the cut-off (Λ). The fixed point – which describes the location of the critical phase transition – is itself a point of scale invariance as it is unaffected by RG transformations.[16] Thus at the fixed point the only elements which are relevant and contribute to the behaviour at the fixed point are those in the renormalisable Hamiltonian. All other terms which may be added to that Hamiltonian will consequently be irrelevant or marginally relevant (see §3.1).

The next section will explore the extent to which each RG approach can be considered to explain the universality of critical phenomena.

3 Universality Explained?

Universality is explained if we are able to show that each member of each universality class has features in common and that those features are sufficient to generate the universal behaviour. In this section I build upon the details of physics given thus far. I argue that the momentum-space explanation is adequate (§3.1) but that the real-space explanation is inadequate (§3.2).

My claims here follow those of Mainwood (2006, pp. 152-187) who argues that the real-space and momentum-space approaches should be distinguished when assessing the RG explanation of universality. Mainwood claims that the real-space approach fails to provide an adequate explanation because the RG transformation needs to be tailored to each model un-

[16] This corresponds physically to the divergence of the correlation length in critical systems.

der consideration – which follows from the non-renormalisability of the Hamiltonians used. As such he considers the real-space approach inadequate to the identification of common features between members of the same universality class.

I suggest that the real-space approach cannot explain universality for a more basic reason: it fails to model the diverse range of systems which fall into the same class and thus does not demonstrate a flow of different systems into the same fixed point; I discuss this further in §3.2. Mainwood's claims bolster my own to the extent that even were the real-space RG to model each distinct system one would still have grounds for doubting the explanation of universality.

In addition to my worries about the real-space RG, I argue that much of the literature mischaracterises the momentum-space explanation of universality: it is commonly implicitly claimed that the explanation demonstrates the irrelevance of the heterogeneous details of physical systems. E.g.:

> The distinct sets of inflowing trajectories reflect their varying physical content of associated irrelevant variables and the corresponding non-universal rates of approach to the asymptotic power laws dictated by $\mathcal{H}$.
>
> [Fisher (1998)]

Similar arguments can be found in (e.g.) Batterman (2014), Kadanoff (2013) and various textbooks. Such arguments are often represented pictorially, see figure 1. This explanatory sketch implies that we are able to include irrelevant details of diverse physical systems in the mathematical representations. However such representations have not been derived for the systems of interest. On the other hand such derivations are not required to explain universality. An explanation is adequate if it exposes features shared by the various systems in the same class, and demonstrates that such features are sufficient for the common behaviour. In short: the literature characterisations of the explanation imply that we can write down irrelevant operators for systems of interest, but we do not know how to do this.

In §4.1 I employ the possibility (and likely availability) of explanatory strategies like those to which the literature refers to argue that, *pace* Batterman (2014) and Morrison (2014), a reductive explanation of universality is achievable.

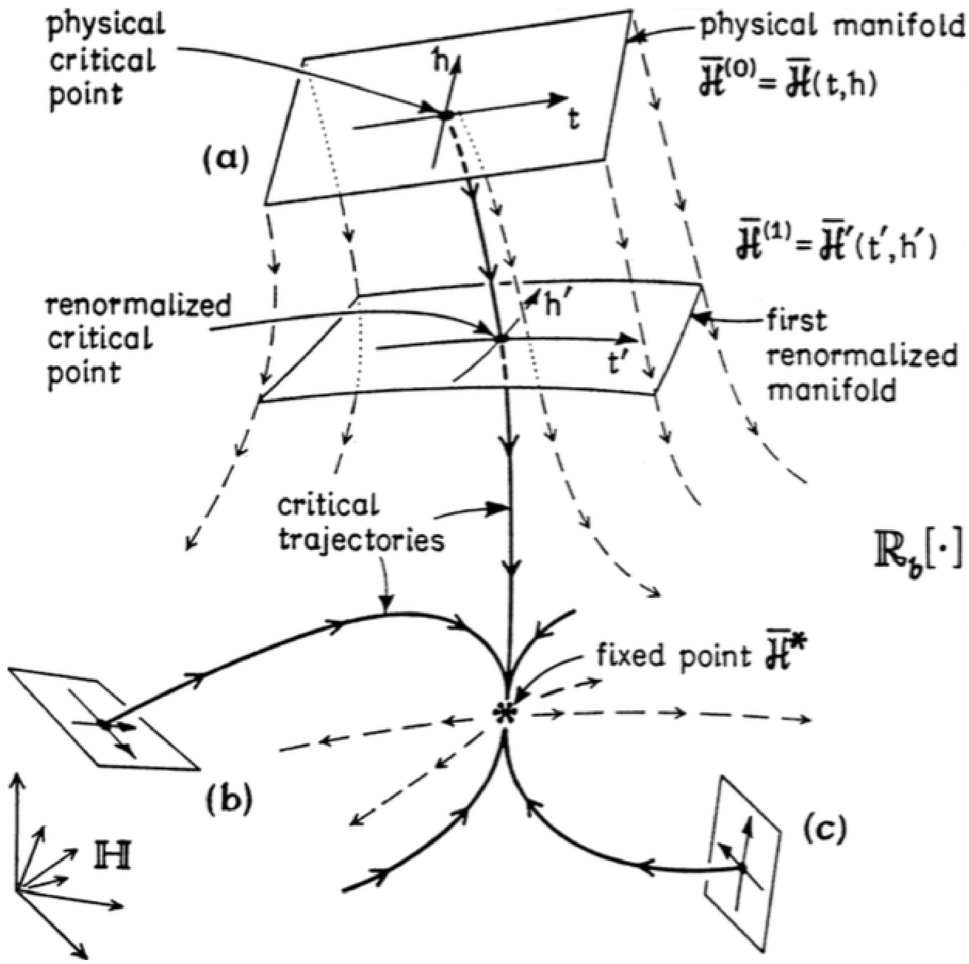

Figure 1: The RG flow in the abstract space of Hamiltonians (or, more precisely, the space of couplings for a fixed Hamiltonian form). Figure from Fisher (1998).

3.1 Momentum-space RG and Crossover

We have sound theoretical reasons to think that the LGW Hamiltonian represents a wide range of physical systems. The physical analysis behind this claim is the renormalisability of the LGW Hamiltonian and the demonstration that certain classes of operators are irrelevant, as discussed on p.12. As summarised by Binney et al. (1992, p.366):

> to the accuracy of our calculation we have shown that any three-dimensional physical system whose Hamiltonian can be written as an even functional of a one-component scalar field should have the same critical behaviour as the Landau-Ginzburg model.

Thus we need an additional justification for each system of interest that its order parameter can be written as a one-component scalar field. Paying close attention to the order parameter of each system in the same class will also ground the various assignations of systems to different universality classes. The order parameter accounts for the symmetry group (i.e. the n of the n-vector model) and the dimensionality. Defining the order parameter for a condensed matter system is not a straightforward process. It depends subtlely on the kind of phase transition the systems undergoes, and which macroscopic features change at such a phase transition. Footnote 11 provides some examples of various order parameters.

The question remains: is universality thus explained? That is, have common features sufficient for common behaviour been identified? The claim that the momentum-space RG approach explains universality depends on the connection between the generality of the LGW Hamiltonian and the individual systems to which it applies. This connection is rooted in the mathematical representation of the various members of the same class, and the RG demonstration that these representations all share the same relevant operators. This line of reasoning thus implies the possibility of representing the features which distinguish such systems.

In order to represent such distinguishing features we must derive operators which are formally irrelevant to critical behaviour. Such operators are not generally known for systems of interest. But the justification for the general applicability of the LGW Hamiltonian assumes that they may be derived, see p.12.

It is possible to derive a correspondence between certain operators and the details of physical systems. This, in combination with the minimal level of detail required to derive the LGW Hamiltonian, provides grounds for our acceptance of the momentum-space approach's explanation of universality. However, the operators for which such a correspondence can be shown are not irrelevant. These are relevant or marginally relevant operators which are pertinent in physical discussions of crossover phenomena. Nonetheless such correspondences help to establish that operators may play the required role in the RG explanation of universality.

Systems undergoing crossover display critical behaviour characteristic of some universality class as they approach T_c, but under repeated iterations of the RG transformations (read: as the temperature moves closer to T_c) they deviate from that behaviour and *cross over* to a different universality class. For example a system near the Heisenberg fixed point may have an additional relevant operator, we might thus define a Heisenberg type ($n = 3$) Hamiltonian including operators for isotropic and anisotropic couplings. It turns out that a system so described will cross over to Ising-type behaviour; for further details see Fisher (1974) and Cardy (1996).

For most instances of universality we have yet to discover irrelevant operators which are physically interpreted as representing those features which distinguish multiple members of the same class. The phenomenon of crossover does suggest that such differences can be modelled. This in turn grounds the claim that the momentum-space approach explains the universality of critical phenomena: it identifies shared features in our systems of interest (represented by the LGW Hamiltonian) sufficient to predict their display of the critical exponents. The expanded Hamiltonians with the irrelevant operators, together with the flow induced by the RG, may be depicted as in figure 1 and thus explain universality

3.2 Real-space RG and the Lattice gas

The real-space RG may be understood by appeal to simple diagrams like that in figure 2. It is thus unfortunate that, as I argue in this section, the explanation provided by the real-space RG is insufficient to account for universality.

The real-space approach allows for the derivation of critical exponents consistent with empirical observation for various models. Furthermore we

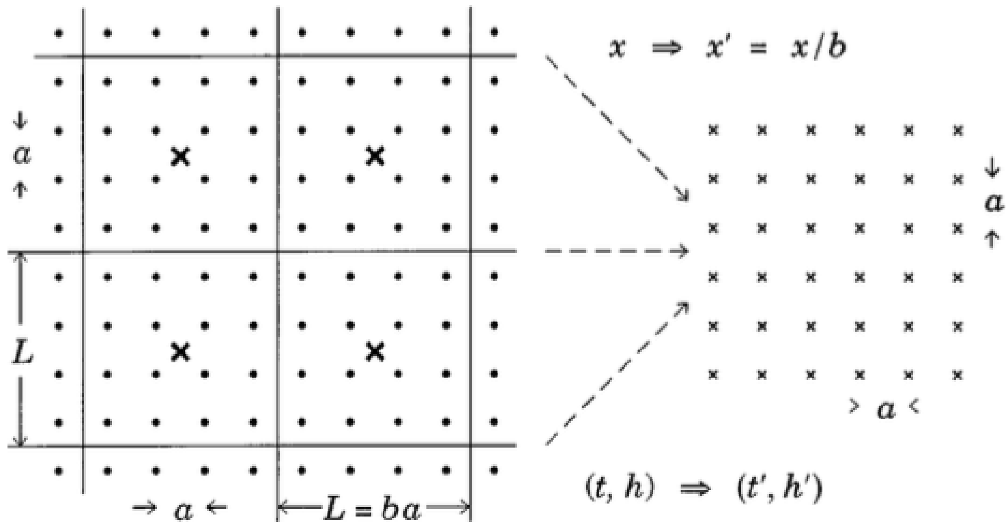

Figure 2: This demonstrates a single application of the real-space RG where a block of spins is replaced by a single larger spin. Figure from Fisher (1998).

have an account of relevance and irrelevance and the claim that: "In general, for fixed points describing second-order critical points, there are two relevant parameters: the temperature and the field conjugate to the order parameter (for the magnet it is the magnetic field)" (Cheung (2011, p.51)). Why is this explanation of universality not sufficient?

The problem with the real-space RG approach is that one has first to assign a model, then apply the RG technique to get to the critical exponents. In general the applicability of the model to each member of the class is not justified. As such universality is assumed rather than explained: no common feature has been identified between the diverse members of the same class. In fact most members are not given a mathematical representation at all. To relate this to the discussion in the last section: the real-space RG explanation is lacking because it cannot classify in general terms putative additions to the n-vector Hamiltonian (equation (2)) as relevant or irrelevant, and thus cannot justify the claim that the n-vector Hamiltonian represents features shared by all members of the same class.

The lattice-gas analogy exemplifies a possible mapping which may supplement and render adequate the real-space explanation. However this supplemented explanation does not explain why liquid-gas systems behave like anisotropic magnets in the critical region but not outside that region. Furthermore it is not generalisable to other examples of various systems falling into the same universality class. In both respects the momentum-space approach outdoes the real-space approach even with the lattice-gas analogy.

The lattice gas model is summarised as follows:

> Consider the Hamiltonian
>
> $$\mathcal{H} = -4J \sum_{\langle ij \rangle} \rho_i \rho_j - \mu \sum_i \rho_i, \tag{10}$$
>
> where $\rho_i = 0, 1$ depending if the site is empty or occupied, and μ is the chemical potential. If we define $\sigma_i = 2\rho_i - 1$, we reobtain the Ising-model Hamiltonian with $B = 2qJ + \mu/2$, where q is the coordination number of the lattice. Thus, for $\mu = -4qJ$, there is an equivalent transition separating the gas phase for $T > T_c$ from a liquid phase for $T < T_c$.
>
> [Pelissetto and Vicari (2002, p.554)]

This mapping is clear enough, but merely shifts the burden of justification. As Pelissetto and Vicari acknowledge "The lattice gas is a crude approximation of a real fluid" (ibid.). Their justification for this approximation is empirical: "Nonetheless, the universality of the behavior around a continuous phase-transition point implies that certain quantities, e.g., critical exponents ... are identical in a real fluid and in a lattice gas, and hence in the Ising model." The model is provided the following rationale in the context of its original presentation:

> The question naturally arises as to the relationship between a lattice gas and a real gas in which the atoms are not confined to move on lattice points. If one replaces the configurational integral in the partition function of the real gas by a summation over lattice sites, one would obtain the partition function of the lattice gas. Theoretically speaking, by making the lattice constant smaller and smaller one could obtain successively better approximations to the partition function of the real gas.
>
> [Lee and Yang (1952, p.412)]

This is rather odd. Although gases may often be modelled as continuum gases, this is itself an idealisation which requires a physical justification. Furthermore, the problem with the application of the Ising model to a physical gas is not that the Ising model is discretised – we expect gases to contain finitely many particles. Rather one should be concerned that the molecules have far more degrees of freedom available to them than the components of uniaxial magnets. The move towards continuum is an idealising step: we sought a de-idealisation to justify the application of the Ising model to liquid-gas systems.

If we were to accept this justification of the lattice-gas model further questions would be raised: for magnets and liquid-gas systems do not display the same behaviour away from the critical point. It is precisely because the systems behave so differently much of the time that universality is startling. Thus, even if it turns out that the lattice-gas analogy gives a good account of liquid-gas systems, additional details are needed to explain the limited applicability of the Ising model to such systems.[17]

Do we have an explanation why these different systems undergo similar behaviour near the critical point? We are told that most of a system's features are irrelevant to its critical behaviour. It turns out, and this is surprising and interesting, that uniaxial magnets and fluids have some behaviour which is approximately described by the same model: namely the Ising model. But this result is a consequence of careful mapping between the systems; it was not an RG result. While a lattice gas model may explain universality to some degree, a generalisable RG explanation is not available on the real-space approach: the RG was used for the derivation of the critical exponents from the models, not in the justification of the applicability of the models to various physical systems.[18]

On the real-space approach we only have an account for what's in common between systems with diverse microphysics when we have a well-motivated mapping between the Ising models and a model for the system in question. The lattice-gas analogy *may* provide one such mapping. How-

[17] Even at the critical point Vause and Sak (1980) argues for a failure of the lattice-gas analogy: while magnets display a symmetry under global spin inversion (in the absence of an external magnetic field) which implies a symmetry of the magnetisation-temperature curve about the temperature axis, liquid-gas systems will not display analogous symmetries of the density-temperature curve.

[18] My arguments do not show that the real-space RG is useless, it is very effective at deriving critical exponents. However, it does not provide an explanation for the phenomenon that various different systems manifest the same exponents.

ever the real-space RG does not allow for a generalised explanation of universality because it cannot underwrite the flow of various different systems into the same fixed point.

4 On 'The Universality Argument Against Reductionism'

So far we have established that the two approaches to the RG ought to be distinguished because the momentum-space approach provides an adequate explanation of the RG while the real-space approach does not. This allows me to address a further controversy: can universality be explained reductively? Batterman claims that a reductive approach lacks the resources to address the question:

> **MR**: How can systems that are heterogeneous at some (typically) micro-scale exhibit the same pattern of behavior at the macro-scale?
> [Batterman (2014, p.8)]

> This [RG response to **MR**] involves mathematical techniques that do not look anything like reductionists' conception of derivation or deducibility. These techniques allow one to show how details that genuinely distinguish realizers from one another are irrelevant to the existence of the pattern.
> [ibid. p.21]

As we have seen, Batterman considers the renormalisation group explanation of **MR** to be adequate, however he understands this explanation to operate exclusively at a level of some abstraction. Batterman claims that a reductive explanation of universality is unavailable. Similarly Margaret Morrison accepts that universality can be explained but she believes any RG explanation to involve the input of information about some scale above the micro. As such she also, though for slightly different reasons, challenges the possibility of a reductive explanation of universality:

> While a reduction in degrees of freedom is an important goal [in the context of explaining universality], it is only part of the

> story. The way this reduction is achieved makes apparent how RG functions not only as a calculational tool but as the source of physical information as well. The latter is accomplished by showing how the process relies on the transformation of structural features of systems (the Hamiltonian in SM, the evolution map for discrete dynamical systems, etc.) rather than specific values for microscopic parameters.
> [Morrison (2014, p.1155)]

In this section I rebut these claims. I argue that the momentum-space RG explanation of universality looks to be reducible, and that there is no good reason to think that such a reduction would fail. Of course, as in most examples of putative reductions in physics (or science more generally) the reduction has not been fully worked out, but I offer plausibility arguments that a reduction is possible based on my claims in §3.1.

Batterman contends that even were physics to provide a complete derivation of each system's critical behaviour from the bottom-up, this would not be sufficient for an explanation of universality. Reductionism is thus, in the context of universality, challenged quite differently from the challenge of infinitary idealisations discussed with respect to first order and continuous phase transitions; see e.g. Batterman (2005) and Batterman (2011).[19] However, the challenge of infinitary idealisations still remains: arguably one cannot define the critical point without going to the infinite limit – discussion of this is beyond the scope of this paper.

Batterman situates the anti-reductionist worries which are presently of interest in a long running debate in the philosophy of science. He uses the case of universality to criticise Sober (1999)'s response to Fodor (1974) and Putnam (1980). Fodor has long argued that multiple realisability – the higher level commonality of multiple microphysically distinct systems – cannot be explained reductively. By contrast Sober argues for pluralism about explanation:

> Generality is one virtue that an explanation can have, but a distinct – and competing – virtue is depth, and it is on this dimension that lower-level explanations often score better than

[19] Batterman (2013) also raises a claim to emergence similar to that in the universality context: there he emphasises that the RG is superior to reductionist modes of explanation because it takes into account fluctuations and thus operates at the 'meso-scale'.

> higher-level explanations. The reductionist claim that lower-level explanations are *always* better and the antireductionist claim that they are *always* worse are both mistaken.
>
> [Sober (1999), p.560]

Sober argues that multiple realisability need not trouble the reductionist. He does so by observing that different kinds of explanation are useful or applicable to different ends. He observes that adding content to an explanation, e.g. by effecting an explanatory reduction, does not stop its being an explanation and that reductive explanations will generally be of interest even if the higher level explanations are adequate in some contexts.

Batterman counters Sober by purporting to provide an explanation which would be undermined by a reduction: the RG explanation of universality. He argues that universality is an instance of multiple realisability where the critical exponents are multiply realised by the various members of their universality class. He claims that the full explanation of universality necessarily proceeds at the higher level in terms of the flow towards the fixed point, and that this should be treated as a paradigm explanation of multiply realised phenomena.

As such, Batterman seeks to undermine Sober's pluralism. It is not that both lower-level and higher-level explanations will do for understanding cases of multiple realisability in different contexts. In fact specifically for certain such cases the higher-level explanations are the only ones which work; the lower-level explanations fail. This failure of explanatory reducibility is due to the need to invoke the RG which, in Batterman's view, must proceed at the higher level.

In response I claim below that, although the RG methods are complex and involve both abstractions and change of variables, the explanations may be seen to derive their impetus from the pairing of physical systems with microphysically derived Hamiltonians. As such, I reject the claims of both Batterman and Morrison that the workings of the RG undermine prospects for achieving the goals of the reductive project.

The upshot of my analysis is that the momentum-space RG does not bring new information to the microphysical descriptions of physical systems.[20] Rather it is a mathematical tool which brings out existing facts

[20]However the real-space RG requires the truncation of the Hamiltonians to which it's applied and may thus be seen to be a source of information as Morrison suggests.

about the Hamiltonians to which it is applied. I claim that one ought to view the momentum-space RG approach – the only approach adequate to explain universality – as a mathematically complex way of describing the changes in the micro system. Hence one may view the explanation of universality as identifying the commonalities between the microsystems which lead to shared 'relevant' variables across diverse systems.

4.1 The Explanation Reduced

Where a higher level explanation of universality is available, each stage of the explanation can be shown to depend for its validity on the microphysical description. Even were aspects of the description to remain irreducible, I claim that those critical to the explanation will be properly understood as claims about the lower level. As such, I undermine Batterman's critique of Sober, and reinstate the possibility that any case of multiple realisability may be explained reductively.

It was shown above (§3.1) that the momentum-space RG explanation of universality implies that operators which represent their heterogeneity could be assigned to each system in the same universality class: the 'irrelevant' operators. Here I suggest that if such irrelevant operators can be provided then they will establish the relationship between the microphysics and the higher level picture and the explanation of universality may be thus reduced. To that end I recapitulate the explanation I discussed in §3.1.

1. Define the effective Hamiltonian for your system of interest:
   - (i) Specify the order parameter with symmetry and dimensionality.
   - (ii) Specify operators in addition to the terms in the LGW Hamiltonian.
2. Apply the RG transformations to that Hamiltonian.
3. Examine the flow towards fixed points in the critical region and note that some operators are irrelevant to the critical behaviour.
4. Thus divide the set of operators into subsets: 'relevant', 'irrelevant' and 'marginally relevant'.
5. Repeat for other systems of interest.

Universality is explained where we identify a commonality among systems in the same universality class sufficient to account for the appearance of common features. This commonality is provided by the LGW Hamiltonian and the order parameter (1.(i)), but its generality arises from the irrelevance of those operators which would distinguish different systems in the same class. As such, if the explanation is adequate then we should, in principle, be able to derive 1.(ii).

How might we go about reducing this explanatory schema? We need to translate the macro-level objects into microphysical terms. It will then be apparent that the facts which ground the explanation of universality are microphysical facts. In addition the effective dynamics induced by the RG transformations must be shown not to introduce novel, higher level facts (*pace* Morrison). Overall it will be shown (*pace* Batterman) that the explanation of universality indeed involves deriving facts about each system in a given universality class from the relevant Hamiltonian.

So there are two stages to this explanatory reduction. Firstly we need to argue that the effective Hamiltonian is simply an abstraction, akin to a change in variables and a loss of detail from the microphysical Hamiltonian. This effective Hamiltonian should depend upon the features of microscopic interactions and should not lose details which specify how the heterogeneous members of the same class differ. Secondly we need to consider the way in which the RG works. Does it bring in new information? Does it exclusively operate at the abstract level?

The first stage can be achieved by looking at derivations of the LGW Hamiltonian. There are many of these, of which I sketched just one in §2.2. There it can be seen that this does depend on features of microscopic Hamiltonians. As seen in §2.2 the justification for the generality of the LGW Hamiltonian relies on the demonstrable irrelevance of operators which distinguish the systems in our universality class.

The irrelevant operators, if they are sufficient to capture the heterogeneity of members of the same universality class would confirm our confidence in the explanation of universality. By the same token such operators would allow the full Hamiltonian to be derived from microscopic details of the systems in question. As argued above, the explanation of universality by the momentum-space Hamiltonian implies that such operators could in principle be written down. This plays a crucial role in providing a reductive explanation of universality. In order further to establish these results one requires knowledge of the irrelevant operators and their microphysical derivations to be spelt out. Of course I have not presented a reduction of the physics of universality; but I have argued on the basis of known physics that such a reduction is possible.

Which common features of the systems of interest ground their similar behaviour? The relevant operators of the LGW Hamiltonian are shared by members of the same universality class, in addition the symmetries and dimensionality of the order parameter represent common features. As can be seen in footnote 11 each system has a different definition for its order parameter. The order parameter for the Ising model is defined in terms of the spins in its microphysical Hamiltonian. Where we have mappings between the various models, such as the lattice-gas analogy we can define the order parameter in terms of the microscopic constituents of the system of interest: where the spins are analogous to occupation number, the sum of spins over a region of the lattice will represent the local magnetisation or, analogously, the local density of the system.

But I've just criticised the lattice gas analogy (see §3.2), I seem to be having my cake and eating it! I criticised the lattice-gas analogy in the context of the real-space RG approach but I accepted that it may provides the basis for a limited real-space explanation of universality. A mapping of this sort is appropriate to the identification of common features between systems of interest: it grounds the common symmetry and dimensionality of order parameters in terms of non-trivial similarities of microphysical structure. It is thus that 1(i) of the momentum-space RG explanation may be reduced. We have here the sketch of an explanatory reduction. I have shown that such an explanation is plausible with respect to point 1 of the enumerated procedure above (p.24).

Before that conclusion can be drawn I ought to discuss the application of the RG itself. It is here that Batterman and Morrison emphasise the irreducibly abstract nature of the explanation on offer. One might think the RG explanation must proceed abstractly because it is often described in terms of a flow through a space parametrised by coefficients of the terms in an abstracted Hamiltonian. One reads the critical exponents off the change in values of these various parameters along the lines of RG flow; see figure 1.

Although this presents a nice way of understanding the function of the RG, the space in question is not essential to the RG itself. "The RG" refers to a collection of transformations which serve to change the length scale of, in this context, the interactions of parts of a physical system described by a Hamiltonian. As such we can use the RG to understand how various properties of the system change as the characteristic length scale of fluctuations increases. Consequently we may derive the RG flow towards the fixed point.

The momentum-space RG apparatus does not, *pace* Morrison, introduce novel information into the Hamiltonian of a system of interest. Central to the RG explanation of universality is the observation that certain properties of a given system will be relevant and others will be irrelevant to its critical behaviour. The RG brings out the relations between various physical properties such as that between the temperature and the correlation length. The pertinent information is all in the microphysical Hamiltonian from which the higher level Hamiltonian is constructed.

While this may seem a strong claim, the onus is on those who take the RG transformations to be more than mere mathematical operations. While the RG is instrumental in bringing to light commonalities in behaviour of diverse systems, to claim that this implies a failure of reduction would be to beg the question.

Morrison's worry is that the application of the RG involves consideration of a family of systems related one to the other by RG transformations. Thus the critical exponents are derived from the family of Hamiltonians rather than from the initial Hamiltonian. While her explication of the RG is, broadly, in line with claims I have made (though she also fails to distinguish momentum-space from real-space approaches), I disagree with Morrison's analysis of why the RG works. It is not that each increasing length scale with its own Hamiltonian brings in new information. Rather each higher scale (renormalised) Hamiltonian may be derived from the initial Hamiltonian. The momentum-space RG doesn't bring in new information, it brings out information about the Hamiltonian: namely the behaviour of systems as the length scale varies.

The various higher level Hamiltonians plausibly depend on microphysical Hamiltonians. And the RG transformations make apparent structural features of those microphysical Hamiltonians. The RG transformations function much like integration: in the former case, presented with a Hamiltonian for a system at some temperature the RG transformations may tell us about physical properties of that system at a temperature closer to the critical point; in the latter case given a force function for a system at a time integration may tell us the velocity of that system at a later time. They are both mathematical procedures which, though often difficult to carry out, bring to light facts about some mathematical object of interest.

The arguments in this section seek to establish that the momentum-space explanation of universality may be reduced at each step. They advert to a particular deflationary way of understanding the function of the RG and a hefty promissory note for a deeper theoretical understanding of the various systems which share universality classes. It is worth repeating that those who believe that universality is explained at any level by the RG are also obliged to accept that these technical lacunae can be filled.

5 Conclusion

Batterman characterises the RG explanation of universality as follows:

> One constructs an enormous abstract space each point of which might represent a real fluid, a possible fluid, a solid, etc. Next one induces on this space a transformation that has the effect, essentially, of eliminating degrees of freedom by some kind of averaging rule.
>
> ... Those systems/models (points in the space) that flow to the same fixed point are in the same universality class–the universality class is delimited–and they will exhibit the same macro-behavior. That macro-behavior can be determined by an analysis of the transformation in the neighborhood of the fixed point.
>
> [Batterman (2014, pp. 13-14)]

This characterisation is representative of the way many physicists and philosophers discuss universality. In this paper I have demonstrated that there are details of this picture which have not been worked out. While we have reasons to be optimistic that these gaps can be filled we ought to acknowledge that this work has not yet been done. I further argued that such an acknowledgement will have consequences for our understanding of the explanation of universality.

I claimed above that there are two ways to cash out this explanation in terms of the available physics and that these ways differ with respect to their success at explaining universality. The real-space approach starts with a model and derives the critical exponents on the basis of that model. It is difficult to see how this approach adequately explains the phenomenon that heterogeneous systems have identical critical behaviour or grounds the picture of the converging RG flows.

The momentum-space approach, on the other hand, explains universality by positing an effective Hamiltonian and deriving the critical exponents from that. That this Hamiltonian is demonstrably general grounds the explanation of universality. Thus the primary moral of my paper is that these two approaches ought to be distinguished

My arguments for this first moral bring clarity to the structure of the explanation of universality. This brings me to my second moral: that the momentum-space explanation of universality may be reduced to a smaller scale physical description. I argued for this claim by outlining how a reduction of the explanation might go and by countering Batterman (2014) and Morrison (2014). The upshot here is that we have on offer a paradigm case of multiple realisability which may be given a reductive explanation. In §4 I briefly outlined the contribution that this may make to an existing literature.

Further work ought to address other anti-reductionist claims in the context of Batterman's discussion of critical phenomena. These pertain to the RG description of the critical region and the fixed point at which certain values diverge. In addition irrelevant operators ought to be defined which correspond to properties of real physical systems. This would supplement the momentum-space explanation of universality and bolster my claim that such an explanation can be reduced.

Acknowledgements

I would like to thank Jeremy Butterfield, Roman Frigg, Eleanor Knox, Mauricio Suarez, as well as audiences at the Oxford University Graduate Philosophy of Physics Seminar, BSPS 2015 and IPP4 for useful comments.

References

Batterman, Robert W. (2005). "Critical phenomena and breaking drops: Infinite idealizations in physics". In: *Studies In History and Philosophy of Science Part B: Studies In History and Philosophy of Modern Physics* 36.2, pp. 225–244.

— (2011). "Emergence, singularities, and symmetry breaking". In: *Foundations of Physics* 41.6, pp. 1031–1050.

— (2013). "The Tyranny of Scales". In: *The Oxford Handbook of Philosophy of Physics*. Ed. by Robert W. Batterman. Oxford University Press, pp. 256–286.

— (2014). "Reduction and Multiple Realizability". In: URL: http://philsci-archive.pitt.edu/11190/.

Binney, J.J. et al. (1992). *The Theory of Critical Phenomena: an Introduction to the Renormalization Group*. Oxford: Clarendon Press, Oxford. ISBN: 0198513941 (hb).

Butterfield, Jeremy and Nazim Bouatta (2011). "Emergence and reduction combined in phase transitions". In: *arXiv preprint arXiv:1104.1371*.

Callender, Craig and Tarun Menon (2013). "Turn and Face the Strange ... Ch-ch-changes Philosophical Questions Raised by Phase Transitions". In: *The Oxford Handbook of Philosophy of Physics*. Ed. by Robert W. Batterman. Oxford University Press, pp. 189–223.

Cardy, John (1996). *Scaling and renormalization in statistical physics*. Cambridge University Press.

Cheung, Anson (2011). "Phase Transitions and Collective Phenomena". Cambridge University Lecture Notes.

Fisher, Michael E (1974). "The renormalization group in the theory of critical behavior". In: *Reviews of Modern Physics* 46.4, p. 597.

Fisher, Michael E (1998). "Renormalization group theory: Its basis and formulation in statistical physics". In: *Reviews of Modern Physics* 70.2, p. 653. Figures reproduced with permission of AMERICAN PHYSICAL SOCIETY in the format republish in a journal/magazine via Copyright Clearance Center.

Fodor, Jerry A (1974). "Special sciences (or: the disunity of science as a working hypothesis)". In: *Synthese* 28.2, pp. 97–115.

Goldenfeld, Nigel (1992). *Lectures on phase transitions and the renormalization group*. Vol. 5. Addison-Wesley Reading, MA.

Kadanoff, Leo P. (2013). "Theories of Matter: Infinities and Renormalization". In: *The Oxford Handbook of Philosophy of Physics*. Ed. by Robert W. Batterman. Oxford University Press, pp. 141–188.

Klein, Bill, Harvey Gould, and Jan Tobochnik (2012). *Kinetics of Phase Transitions*. URL: `http://stp.clarku.edu/kleinnotes/`.

Lee, Tsung-Dao and Chen-Ning Yang (1952). "Statistical theory of equations of state and phase transitions. II. Lattice gas and Ising model". In: *Physical Review* 87.3, p. 410.

Mainwood, Paul (2006). "Is More Different? Emergent Properties in Physics". PhD thesis. University of Oxford.

Morrison, Margaret (2014). "Complex Systems and Renormalization Group Explanations". In: *Philosophy of Science* 81.5, pp. 1144–1156.

Niss, Martin (2005). "History of the Lenz-Ising model 1920–1950: from ferromagnetic to cooperative phenomena". In: *Archive for history of exact sciences* 59.3, pp. 267–318.

Pelissetto, Andrea and Ettore Vicari (2002). "Critical Phenomena and Renormalization Group Theory". In: *Physics Reports* 368.6, pp. 549–727.

Putnam, Hilary (1980). "Philosophy and our mental life". In: *Readings in the Philosophy of Psychology* 1, pp. 134–143.

Sengers, Jan V and Joseph G Shanks (2009). "Experimental critical-exponent values for fluids". In: *Journal of Statistical Physics* 137.5-6, pp. 857–877.

Sober, Elliott (1999). "The multiple realizability argument against reductionism". In: *Philosophy of Science*, pp. 542–564.

Stanley, H Eugene (1999). "Scaling, universality, and renormalization: Three pillars of modern critical phenomena". In: *Reviews of modern physics* 71.2, S358–S366.

Vause, C. and J. Sak (1980). "Non-Ising-like effects in the liquid-vapor transition: Equations of state". In: *Phys. Rev. A* 21 (6), pp. 2099–2114.